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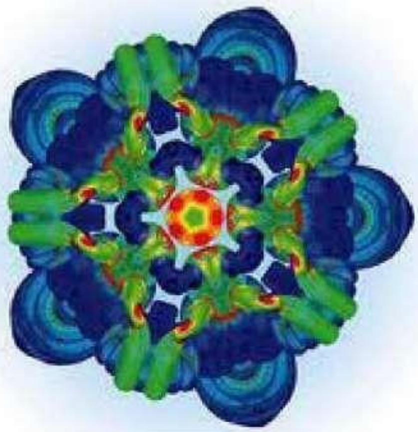

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Determinant for Non-Square Fuzzy Matrices with Compatible Norm

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***Abstract**: In this paper Determinant for Non-Square Fuzzzy Matrices and its properties are studied. Using elementary operations. Some important algebraic properties of addition, Scalar Multiplication of Determinant for Non-Square Fuzzzy Matrices are discussed. A new compatible Norm ‖. ‖c is defined and special type of Non-Square Fuzzy Matrix multiplication are used.*

***Keywords**: Fuzzy Matrix $\mathcal{F}_{mm}$, Determinant for Non-Square Fuzzy Matrices (NSFM), compatible Matrices, compatible Norm‖. ‖c .*

***2010 AMS Subject Classification:** 03E72, 15A15, 15A60*

I. Introduction

The concept of Fuzzy set was introduced by Zadeh [8] A.Arunkumar, S.Murthy, G.Ganapathy [1] introduced Determinant For Non-Square Fuzzy Matrices. In 1995 Ragab.M.Z and Eman [2] introduced the determinant and adjoint of Square Fuzzy Matrices. Nagoorgani.A and Kalyani.G.[3] Introduced the Binormed sequences in fuzzy matrices .Nagoorgani A. and Manikandan A.R. [4] Introduced on Fuzzy Determinant norm Matrices. AR.Meenakshi [5] introduced some concept of matrices theory and applications in Fuzzy Matrices. Dennis .Bernstein [6] introduced compatible norm in Matrix Mathematics Theory, Facts and Formulas. A.K. Shymal and Madhumangal Pal [7] properties of triangular Fuzzy matrices. In this paper, the Concept NSFM with Compatible norm is discussed. In section 1, some basic definitions and properties are given. In section 3, the purpose of introduction Determinant for NSFM are explained in $\mathcal{F}_{mm}$. In section 4, some properties of NSFM are given .In section 5, compatible norm used for two NSFM multiplication.

II. Preliminaries

We consider $\mathcal{F} = [0,1]$ the fuzzy algebra with operator $[+,\cdot]$ and the standard order‘ $\leq$ ” where $a + b = max\ \{a,\ b\ \}, a \cdot b = min\ \{a,\ b\}$ for all a,b in $\mathcal{F}$. $\mathcal{F}$ is a commutative semiring with additive and multiplicative identities 0 and 1 respectively . Let $\mathcal{F}_{mm}$ denote the set of all $m \times m$ NSFM over $\mathcal{F}_{mm}$. In short $\mathcal{F}_{mm}$ is the set of NSFM of order$m \times m$ define “ $+$ ” and Scalar Multiplication in $\mathcal{F}_{mm}$ as $A + B = [a_{ij} + b_{ij}]$ where $A = [a_{ij}]$ and $B = [b_{ij}]$ and $cA = [ca_{ij}]$
where c is in $[0,1]$ with these operations $\mathcal{F}_{mm}$ Forms a linear space. NSFM Multiplication is number of column in the first Matrix must be equal to the number of rows in the second matrix with these operations $\mathcal{F}_{mm}$ forms a linear space.

III. Determinant For Non-Square Fuzzy Matrices

(i) Definition :
An $m \times m$ matrix $A = [a_{ij}]$ whose components are in the unit interval $[0,1]$ is called fuzzy matrix.

(ii) Definition :
The determinant $|A|$ of an $m \times m$ fuzzy matrix A is defined as follows; $|A| = \sum_{\sigma \in S_n} a_{1\sigma(1)} a_{2\sigma(2)} \dots a_{n\sigma(n)}$ Where Sn denotes the symmetric group of all permutations of the indices $(1,2 \dots .n)$.

(iii) Definition :
A Non-Square fuzzy matrix [NSFM] $A = [a_{ij}]$ of order $m \times m$ over$\mathcal{F}_{mm}$ If $m >$ m then the matrix A is called horizontal Non-Square fuzzy matrix. Otherwise A is called Vertical Non Square fuzzy matrix.

(iv) Definition :

To every Non-Square fuzzy matrix [NSFM] $A = [a_{ij}]$ of order $m \times m$ over $\mathcal{F}_{mm}$ with entries as unit interval [0,1] Determinant $|A|$ of $m \times m$ over $\mathcal{F}_{mm}$ fuzzy matrix A is defined as follows. $|A| = \sum_{\sigma \in S_n} a_{1\sigma(1)} a_{2\sigma(2)} \ldots a_{m\sigma(n)}$ (where Sn denotes mm).

Case(i): If $A = [\, a_{11} \; a_{12} \; a_{13} \ldots . a_{1n}]$ then its $|A| = a_{11} \; a_{12} \; a_{13} \ldots . a_{1n} = \sum_{i=1}^{n} a_{1i}$

Case (ii):

If $A = \begin{bmatrix} a_{11} \\ a_{21} \\ \ldots \\ a_{m1} \end{bmatrix}$ then its $|A| = a_{11} \; a_{21} \ldots . a_{m1} = \sum_{i=1}^{m} a_{i1}$

Case (iii):

If $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \end{bmatrix}$ then its $|A| = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \begin{vmatrix} a_{1i} & a_{2i} \\ a_{j} & a_{2j} \end{vmatrix}$

Case (iv):

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \ldots & \ldots \\ a_{m1} & a_{m2} \end{bmatrix}$ then its $|A| = \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \begin{vmatrix} a_{i1} & a_{i2} \\ a_{j1} & a_{j2} \end{vmatrix}$

(v) Definition :

The NSFM $|A| = [a_{ij}]$ be the order $m \times m$ over $\mathcal{F}_{mm}$ .If the order $m \times m \geq 3$ .The minor of arbitrary element a_{ij} is the determinant of the value.

(vi) Definition : Non Square fuzzy matrices of minor:

The NSFM A= (a_{ij}) be the order of $m \times m$ over $\mathcal{F}_{mm}$ .The minor of an element aij in det $|A|$is the order $(m-1) \times (n-1)$. NSFM formed by deleting i-th row and the j-th column from $A = (a_{ij})$ denoted by M_{ij} .

(vii) Definition : Cofactor:

The NSFM $A = (a_{ij})$ be the order of $m \times m$ over $\mathcal{F}_{mm}$. The Cofactor of an element a_{ij} is denoted by A_{ij} and is defined as $A_{ij} = (1)^{i+j} M_{ij}$.

(viii) Definition :

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \end{bmatrix}$ then its

Determinant is defined

$|A| = a_{11}M_{11} + a_{12}M_{12} + \ldots\ldots + a_{1n}M_{1n}$

$|A| = \sum_{i=1}^{n} a_{1i} M_{1i}$.

(ix) Definition :

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & a_{m3} \end{bmatrix}$

$|A| = a_{11}M_{11} + a_{21}M_{21} + \ldots\ldots + a_{m1}M_{m1}$

$|A| = \sum_{i=1}^{m} a_{i1} M_{i1}$.

(x) Theorem :

The value of the NSFM determinant |A| = (a_{ij}) be the order $m \times m$ over $\mathcal{F}_{mm}$ unchanged . When we interchanged rows into columns and columns into rows that is $|A| = |A^T|$for any Non-Square Fuzzy matrix A.

Proof:

Consider the Matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

By the definition (viii) of $|A|$we have

$|A| = a_{11}M_{11} + a_{12}M_{12} + a_{13}M_{13} + a_{14}M_{14}$

$= a11\{\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{24} \\ a_{32} & a_{34} \end{vmatrix} + \begin{vmatrix} a_{23} & a_{24} \\ a_{33} & a_{34} \end{vmatrix}\}$

$+ a12\{\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{23} & a_{24} \\ a_{33} & a_{34} \end{vmatrix}\}$

$+ a13\{\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{21} & a_{24} \\ a_{31} & a_{34} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{24} \\ a_{32} & a_{34} \end{vmatrix}\}$

$$+a_{14}\{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} + \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}\}$$

$$= a_{11}\{(a_{22}a_{33} + a_{23}a_{32}) + (a_{22}a_{34} + a_{24}a_{32}) + (a_{23}a_{34} + a_{24}a_{33})\} + a_{12}\{(a_{21}a_{33} + a_{23}a_{31}) + (a_{21}a_{34} + a_{24}a_{31}) + (a_{23}a_{34} + a_{24}a_{33})\} + a_{13}\{(a_{21}a_{33} + a_{23}a_{31}) + (a_{21}a_{34} + a_{31}a_{24}) + (a_{22}a_{34} + a_{24}a_{32})\} + a_{14}\{(a_{21}a_{32} + a_{31}a_{22}) + (a_{21}a_{33} + a_{31}a_{23}) + (a_{22}a_{33} + a_{23}a_{32})\}$$

........1

Let us interchange the rows and columns of A we have

$$|A^T| = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \\ a_{14} & a_{24} & a_{34} \end{bmatrix}$$

By the definition (ix) as defined we have

$$= a_{11}M_{11} + a_{21}M_{21} + a_{31}M_{31} + a_{41}M_{41}$$

$$= a_{11}\{\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{24} \\ a_{32} & a_{34} \end{vmatrix} + \begin{vmatrix} a_{23} & a_{24} \\ a_{33} & a_{34} \end{vmatrix}\}$$

$$+a_{12}\{\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{23} & a_{24} \\ a_{33} & a_{34} \end{vmatrix}\}$$

$$+a_{13}\{\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{21} & a_{24} \\ a_{31} & a_{34} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{24} \\ a_{32} & a_{34} \end{vmatrix}\}$$

$$+a_{14}\{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} + \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}\}$$

$$= a_{11}\{(a_{22}a_{33} + a_{23}a_{32}) + (a_{22}a_{34} + a_{24}a_{32}) + (a_{23}a_{34} + a_{24}a_{33})\} + a_{12}\{(a_{21}a_{33} + a_{23}a_{31}) + (a_{21}a_{34} + a_{24}a_{31}) + (a_{23}a_{34} + a_{24}a_{33})\} + a_{13}\{(a_{21}a_{33} + a_{23}a_{31}) + (a_{21}a_{34} + a_{31}a_{24}) + (a_{22}a_{34} + a_{24}a_{32})\} + a_{14}\{(a_{21}a_{32} + a_{31}a_{22}) + (a_{21}a_{33} + a_{31}a_{23}) + (a_{22}a_{33} + a_{23}a_{32})\}.$$

.........2

From equation (1) and (2) we obtain proof of the theorem.

(A). Example :

If $A = \begin{bmatrix} 0.5 & 0.0 & 0.4 & 0.6 \\ 0.1 & 0.9 & 0.7 & 0.5 \\ 0.8 & 0.3 & 0.5 & 0.2 \end{bmatrix}$

$$|A| = 0.5\{\begin{vmatrix} 0.9 & 0.7 \\ 0.3 & 0.5 \end{vmatrix} + \begin{vmatrix} 0.9 & 0.5 \\ 0.3 & 0.2 \end{vmatrix} + \begin{vmatrix} 0.7 & 0.5 \\ 0.5 & 0.2 \end{vmatrix}\}$$

$$+0.0\{\begin{vmatrix} 0.1 & 0.7 \\ 0.8 & 0.5 \end{vmatrix} + \begin{vmatrix} 0.1 & 0.5 \\ 0.8 & 0.2 \end{vmatrix} + \begin{vmatrix} 0.7 & 0.5 \\ 0.5 & 0.2 \end{vmatrix}\}$$

$$+0.4\{\begin{vmatrix} 0.1 & 0.9 \\ 0.8 & 0.3 \end{vmatrix} + \begin{vmatrix} 0.1 & 0.5 \\ 0.8 & 0.2 \end{vmatrix} + \begin{vmatrix} 0.9 & 0.5 \\ 0.3 & 0.2 \end{vmatrix}\}$$

$$+0.6\{\begin{vmatrix} 0.1 & 0.9 \\ 0.8 & 0.3 \end{vmatrix} + \begin{vmatrix} 0.1 & 0.7 \\ 0.8 & 0.3 \end{vmatrix} + \begin{vmatrix} 0.9 & 0.7 \\ 0.3 & 0.5 \end{vmatrix}\}$$

$|A| = 0.6.$

In a similarily we prove the following properties:

(xi) Theorem :

If any two rows of horizontal NSFM determinant be the order $m \times m$ over $\mathcal{F}_{mm}$ are interchanged, then horizontal NSFM determinant numerical value is unaltered.

(xii) Theorem :

If any two coloumns of vertical NSFM determinant the order $m \times m$ over $\mathcal{F}_{mm}$ are interchanged, then vertical NSFM determinant numerical value is unaltered.

IV. Properties of Determinant for Non-Square Fuzzy Matrices

(xiii) Theorem :

For any three matrices A,B,C are NSFM of the same order $m \times m$ over $\mathcal{F}_{mm}$.The set $\mathcal{F}_{mm}$is a fuzzy vector space under the operations defined as A+B=(max $\{a_{ij}, b_{ij}\}$) , A+B+C = max(a_{ij}, b_{ij}, c_{ij}) and αA = min(α,a_{ij}) ,βB = min(β, b_{ij}) . We have $|A|$=[a_{ij}] , $|B|$=[b_{ij}] ,

$|C|$=[c_{ij}] ∈ $\mathcal{F}_{mm}$ and α,β ∈ $\mathcal{F}$.Since $\mathcal{F}$ = [0,1] ,α,β in[0,1] and [α+β] in [0,1].

Proof:

For NSFM of A,B,C ∈ $\mathcal{F}_{mm}$

Case (i): $|A + B| = |B + A|$ (Commutative Property)

Case (ii): $|A + (B + C)| = |(A + B) + C|$ (Associative Property)

Case (iii): $|A + B| = |A| + |B|$, $|A + B|^T = |A|^T + |B|^T$

Case (iv): $|A + A| = |A|$ (Idompotent)

Case (v): For all NSFM of A∈ $\mathcal{F}_{mm}$, there exists an element 0∈ $\mathcal{F}_{mm}$

Gani and Pappa, American International Journal of Research in Science, Technology, Engineering & Mathematics, Special Issue of 5th International Conference on Mathematical Methods and Computation (ICOMAC – 2019), February 20-21, 2019, pp. 354-359.

$|A + 0| = |A|$ (Universal bound)

Case (vi): For all NSFM of A∈ $\mathcal{F}_{mm}$, there exists an element J∈ $\mathcal{F}_{mm}$

$|A + J| = |J|$ (Universal bound)

Case (vii): $|\alpha A| = \alpha|A|$ For any α in [0,1]

Case (viii): $|\alpha A|^T = \alpha|A|^T$ For any α in [0,1]

Case (ix): $|\alpha(A + B)| = \alpha|A| + \alpha|B|$ For any α in [0,1]

$|\alpha(A + B)| = |\alpha A + \alpha B)|$

$= |\alpha A| + |\alpha B|$

$= \alpha|A| + \alpha|B|$

Case (x): $|(\alpha + \beta)A| = \alpha|A| + \beta|A|$ For all $\alpha + \beta$ in [0,1]

$|(\alpha + \beta)A| = |\alpha A + \beta A|$

$= |\alpha A| + |\beta A|$

$= \alpha|A| + \beta|A|$

Case (xi): $\alpha|\beta A| = \alpha\beta|A|$ For any α, β in [0,1]

$\alpha|\beta A| = |\alpha\beta A|$

$= \alpha\beta|A$

Case (xii):

$|\alpha A + \beta B|T = \alpha|A|T + \beta|B|T$ For all $\alpha + \beta$ in [0,1]

$|\alpha A + \beta B|T = |\alpha A|T + |\beta B|T$

$= \alpha|A|T + \beta|B|T$

V. Determinant For Two Non-Square Fuzzy Matrix Multiplication

(xiv)Definition : (Compatible Non-Square Fuzzy Matrices):

Compatible Fuzzy Matrices which can be multiplayed for this to be possible , The number of columns in the first non-square Fuzzy Matrix must be equal to the number of rows in the second-square Fuzzy matrix must be equal to the number of rows in the second non-square Fuzzy Matrix (NSFM) .the product of $m \ x \ \wp$ Non-square Fuzzy Matix and $\wp x \ m$ Non-Square fuzzy matrix has order $m \ x \ m$ Non-square Fuzzy Matrix over $\mathcal{F}_{mm}$ we consider $\mathcal{F}$ =[0,1].

(xv) **Definition : (Compatible norm $\|.\|_c$):**

Let $(\mathcal{F})_{mm}$ is the set of all $(m \, x \, m)$ NSFM over $\mathcal{F}$ =[0,1] . Define the norms $\|.\|_c$, $\|.\|_c'$, $\|.\|_c''$ on the order $m \, x \, m$, $m \, x \, \wp$, $\wp \, x \, m$ over $\mathcal{F}_{mm}$ respectively, are compatible if for all A∈ $\mathcal{F}_{m\wp}$ and B ∈ $\mathcal{F}_{\wp m}$. Then

$$\| AB \|_c \leq \| A \|_c' \| B \|_c''.$$

(xvi) **Theorem :**

If two NSFM satisfy the compatibility condition then the multiplication of these NSFM will either be square fuzzy matrix or Non-Square fuzzy matrix which depends up on rows and coloumns of the first and second NSFM respectively.

Proof:

We have $\| A \|_c = [a_{ik}]$ and $\| B \|_c = [b_{kj}]$ then $\| AB \|_c = \sum_{j=1}^{n} a_{ik} \, b_{kj}$ where $a_{ik}b_{kj}$=min[a_{ik}, b_{kj}].

(xvii) **Theorem :**

If $\mathcal{F}_{mm}$ is the set of all $m \, x \, m$ NSFM over F=[0,1] then for every A and B compatible in $\mathcal{F}_{mm}$ and any scalar α,β in [0,1] we have

(i) $\|A\|_c \geq 0$ and $\|A\|_c = 0$ if and only if $A = 0$

(ii) $\|\alpha A\|_c = \alpha\|A\|_c$ for any α in [0,1]

(iii) $\|AB\|_c \leq \|A\|_c' \|B\|_c''$ for A, B in $\mathcal{F}_{mm}$

(iv) $\|AB\|_c^T \leq \|B\|_c''^T \|A\|_c'^T$ for A, B in $\mathcal{F}_{mm}$

(v) $\| \alpha(AB) \|_c = \| (\alpha A)B \|_c = \| A(\alpha B) \|_c$ for A, B in $\mathcal{F}_{mm}$ for any α in[0,1]

(vi) $\| \alpha(\beta A) \|_c = \| (\alpha\beta)A \|_c$ A in $\mathcal{F}_{mm}$ for any αβ in [0,1]

Proof:

(i) $\|A\|_c \geq 0$ and $\|A\|_c = 0$ if and only if $A = 0$

If $\|A\|_c$ is a NSFM in $\mathcal{F}_{mm}$ since a_{ij} ∈[0,1] then $\|A\|_c \geq 0$ for all in $\mathcal{F}_{mm}$

If $\|A\|_c = 0$ then $a_{ij} = 0$ for all i and j $A = 0$

Conversely if $A = 0$ then $\|A\|_c$ =0

$\|A\|_c$ =0 if $A = 0$.

(ii) $\|\alpha A\|_c = \alpha\|A\|_c$ for any α in [0,1]

If α in[0,1] then $\|\alpha A\|_c = \alpha\|A\|_c$

$\|\alpha A\|_c = [\alpha \ (a_{ij})]$

$= \alpha[a_{ij}]$

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BALASUNDARAM VANI
02.07.2024 12:42

$= \alpha\|A\|_c$

(iii) $\|AB\|_c \le \|A\|_c' \|B\|_c''$ for A, B in $\mathcal{F}_{mm}$

If $(i,j)^{th}$ entry of $AB = D$ then the entries of D are given by

$d_{ij} = \sum_{k=1}^{p} a_{ik}, b_{kj}$

$d_{ij} = \sum_{k=1}^{p} \{\min(a_{ik}, b_{kj})\}$

$d_{ij} = min\ (a_{i1}, b_{j1}) + min(a_{i2}, , b_{j2}) \ldots \ldots . min(a_{im}, , b_{jn})$

$\|A\|_c = [a_{ij}]$

$\|A\|_c = \sum_{j=1}^{M} a_{1i}, m_{1i}$

$= a_{11}\, m_{11} + a_{12}\, m_{12} + \cdots + a_{1m}\, m_{1m}$

$\|B\|_c = \sum_{j=1}^{M} b_{1i}, m_{1i}$

$= b_{11}\, m_{11} + b_{12}\, m_{12} + \cdots + b_{1m}\, m_{1m}$

$D \le\ min\{[a_{ik}], [b_{kj}]\}$

$\|AB\|_c \le \|A\|_c' \|B\|_c''$ for A, B in $\mathcal{F}_{mm}$

(iv) $\|AB\|_c^T \le \|B\|_c''^T \|A\|_c'^T$ for A, B in $\mathcal{F}_{mm}$

If $(i,j)^{th}$ entry of $(AB)^T = D^T$ then the entries of D^T are given $A^T = a_{ki}$, $B^T = b_{kj}$; $A^T = \wp x m$, $B^T = m x \wp$ by

$d_{ij} = \sum_{k=1}^{p} b_{jk}, a_{ki}$

$d_{ij} = \sum_{k=1}^{p} \min(b_{jk}, a_{ki})$

$d_{ij} = min(b_{i1}, a_{j1}) + min(b_{i2}, a_{j2}) \ldots . . + min(b_{in}, a_{jm})$

$A^T = a_{ki}$ order $\wp x m$ $B^T = b_{jk}$ order $m x \wp$

$\|A\|^T_c = \sum_{j=1}^{M} a_{1i}, m_1 = a_{11}\, m_{11} + a_{21}\, m_{21} + \cdots + a_{m1}\, m_{m1}$

$\|B\|^T_c = \sum_{j=1}^{M} b_{1i}, m_{1i} = b_{11}\, m_{11} + b_{21}\, m_{21} + \cdots + b_{m1}\, m_{m1}$

$D^T = min\{(b_{jk}), (a_{ki})\}$

$\|AB\|_c^T \le \|B\|_c''^T \|A\|_c'^T$ for A, B in $\mathcal{F}_{mm}$

(v)$\|\alpha(AB)\|_c = \|(\alpha A)B\|_c = \|A(\alpha B)\|_c$ α in $[0,1]$, $A = [a_{ij}]$, $B = [b_{ij}]$

$\|\ \alpha(AB)\ \|_c = \alpha\ [a_{ij} b_{ij}]$

$= [\alpha\ a_{ij}\ b_{ij}]$

$= [\alpha\ a_{ij}]\ b_{ij}$

$= \|\ (\alpha A)B\ \|_c$

$= [\alpha\ a_{ij}]\ b_{ij}$

$= [a_{ij}\ \alpha]\ b_{ij}$

$= [a_{ij}\ \alpha\ b_{ij}]$

$= [a_{ij}]\ [\alpha\ b_{ij}]$

$= \|\ \alpha(AB)\ \|_c$

(vi) $\|\ \alpha(\beta A)\ \|\ c = \|\ (\alpha\beta)A\ \|_c$ If α, β in $[0,1]$, $A = [a_{ij}]$

$\alpha(\beta A) = \alpha(\ \beta\ a_{ij})$

$= [\alpha\ \beta\ a_{ij}]$

$= (\alpha\beta)\ (a_{ij})$

$= (\alpha\beta)\ [a_{ij}]$

$= \|\ \alpha(\beta A)\ \|_c$.

(A). Example :

Verify $\|\ AB\ \|_c \le \|\ A\ \|_c'\ \|\ B\ \|_c''$.

If $A = \begin{bmatrix} 0.5 & 0.0 & 0.4 & 0.6 \\ 0.1 & 0.9 & 0.7 & 0.5 \\ 0.8 & 0.3 & 0.5 & 0.2 \end{bmatrix}$ and $B = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.7 \\ 0.8 & 0.5 \\ 0.4 & 0.9 \end{bmatrix}$

$\|\ AB\ \|_c = \begin{bmatrix} 0.4 & 0.6 \\ 0.7 & 0.7 \\ 0.5 & 0.5 \end{bmatrix}$

$= 0.6$

$\|\ AB\ \|_c \le \|\ A\ \|_c'\ \|\ B\ \|_c''$.

$0.6 \le (0.6)(0.8)$

$0.6 \le 0.6$.

(xviii) Theorem :

Let A be a NSFM and A^T be the transpose of A. The multiple of A and A^T is equal to a square fuzzy matrix. Then $\|\ AA^T\ \|_c \ne \|\ A^TA\ \|_c$

Proof:

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$|A| = [a_{ij}]$ $A^T = [a_{ji}]$
$\| AA^T \|_c = \sum_{j=1}^{n} \{\min(a_{ij}, a_{ji})\}$. [Square Fuzzy Matrix]
$\| A^TA \|_c = \sum_{i=1}^{m} \{\min(a_{ji}, a_{ij})\}$. [Square Fuzzy Matrix]

(xix) Theorem :

If n=1, the norms $\|\cdot\|_c$, $\|\cdot\|_c'$, $\|\cdot\|_c''$ on $\mathcal{F}_m, \mathcal{F}_{mp}, \mathcal{F}_p$ respectively, are compatible if for all $A \epsilon \mathcal{F}_{mp}$ and $x\ \epsilon \mathcal{F}_p$ Then $\| Ax \|_c \leq \| A \|_c' \| \bar{x} \|_c''$

Proof:

Let $\| A \|_c'$ be the NSFM $\mathcal{F}_{mm}$ over $\mathcal{F} = [0,1]$ $\| \bar{x} \|_c''$ be the fuzzy norm vector then $[a_{ij}]$ is compatible the fuzzy norm vector $\| \bar{x} \|_c''$ then $\| Ax \|_c \leq \| A \|_c' \| \bar{x} \|_c''$

Let A be $(m\ x\ m)$ NSFM of $\mathcal{F}_{mm}$

$$\|Ax\| = \left\| \begin{matrix} A_{1p} & \bar{x}_P \\ A_{2p} & \bar{x}_P \\ \dots & \dots \\ A_{mp} & \bar{x}_P \end{matrix} \right\| \leq \sum_{p=1}^{n} [a_{mp}] \ \| \bar{x}_p \ \|]$$

$$\leq [a_{mp}] \ \| \bar{x}_p \ \|]$$

$$\| Ax \|_c \leq \|A\|_c' \ \| \bar{x} \|_c''$$

Furthermore, the norm $\|.\|_c$ on $\mathcal{F}_m$ compatible with the norm $\|.\|_c$ on $\mathcal{F}_{nn}$ if for $A\epsilon\mathcal{F}_{nn}$ and $x\ \epsilon\mathcal{F}_n$.

$$\| Ax \|_c \leq \| A \|_c' \ \| \bar{x} \|_c.$$

VI. Conclusion

In this paper new definition for the non-square fuzzy matrices and its properties are discussed in fuzzy environment. A numerical example is given to clarify the developed theory and the proposed non-square fuzzy matrix compaitable norm.

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Steady- state concentrations of carbon dioxide absorbed into phenyl glycidyl ether solutions by Taylor series method

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In this paper, two coupled nonlinear differential equations related to carbon dioxide (CO_2) and phenyl glycidyl ether (PGE) concentrations are solved using the Taylor series method. This model has based a set of mixed boundary conditions for Dirichlet and Neumann. This method yields quick converging, easily computable, and efficiently verifiable approximate closed-form solutions. The effect of the parameters on the enhancement factor is also discussed. The analytical result is programmed using computer algebra packages like Maple. The numerical result is compared with the approximate solutions obtained by this method, and a satisfactory agreement has been noted.

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Graceful labeling of F graph and mirror image of F graph

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A graph G(V,E) is graceful, if there exists an injective map $f:V(G) \rightarrow \{0,1,2,\ldots,q\}$ such that its induced map $f^+:E(G) \rightarrow \{1,2,3,\ldots,q\}$ is defined by $f^+(uv) = |f(u) - f(v)|$ for every edge uv in G where f^+ is injective. A graph G is called graceful if it admits a graceful labeling. In this paper, the graceful labeling for the F graph and its mirror image is obtained.